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FORMATION OF A GAS BUBBLE ON A VIBRATING CAPILLARY IMMERSED IN A LIQUID

I. S. Grachev, D. T. Kokorev,*
and V. F. Yudaev

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The formation of a train of bubbles in a low-viscosity liquid is investigated. The dependence of the gas flow rate during formation of the bubble train on the vibrational acceleration of the capillary is determined.

One of the techniques used to intensify mass transfer at a liquid–gas or liquid–liquid interface is to disperse one of the phases by means of a vibrating nozzle or macrocapillary (bubbling, dispersion, etc.). This technique enables one to control the particle size of the dispersed phase over a wide range and to increase the relative velocity of the interacting phases. In the present study we attempt to formulate a fluid-mechanical description of the formation of a single gas bubble on a vibrating macrocapillary immersed in a liquid.

To derive the bubble-growth equation we assume that: 1) the surface-tension forces impart a spherical shape to the bubble; 2) prior to breakoff the bubble remains rigidly connected to the capillary, which vibrates in a vertical plane according to a harmonic law. In the direction of the vertical axis, therefore, the center of the growing bubble is simultaneously involved in two motions, one reciprocating $A \sin \omega t$ and the other translational $R(t)$ due to its own growth (Fig. 1a), such that

$$z = R(t) + A \sin \omega t. \tag{1}$$

The velocity of the center of the bubble is

$$U = \frac{dz}{dt} = \frac{dR}{dt} + A\omega \cos \omega t. \tag{2}$$

The velocity potential of the liquid surrounding the growing spherical bubble is written as follows [1]:

*Deceased.

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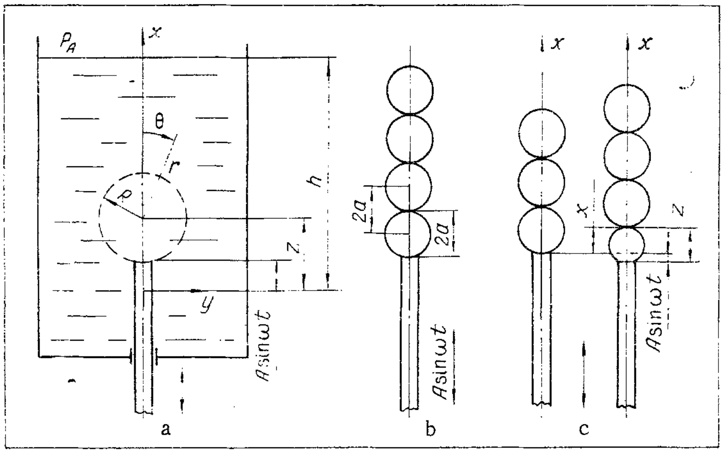

Fig. 1. Diagram for derivation of the growth equation for a spherical bubble on a vertically vibrating capillary (a), bubble-train formation (b), and illustration of buddble formation in a train (c).

$$\Phi = -\frac{R^2}{r}\frac{dR}{dt} - \frac{UR^3}{2r^2}\cos\Theta. \tag{3}$$

We use the Bernoulli equation and determine the pressure at an arbitrary point of the liquid:

$$\frac{P_r}{\rho} = -\left(\frac{\partial\Phi}{\partial t}\right)_{x,y} - \frac{q^2}{2} + \frac{P_A}{\rho} + g\,(h - A\sin\omega t - x). \tag{4}$$

In calculating $(\partial\Phi/\partial t)_{x,y}$ it must be borne in mind that

$$\left(\frac{\partial\Theta}{\partial t}\right)_{x,y} = \frac{U}{r}\sin\Theta; \quad \left(\frac{\partial r}{\partial t}\right)_{x,y} = -U\cos\Theta. \tag{5}$$

The absolute velocity q of the liquid is given by the expression

$$q^2 = \left(\frac{\partial\Phi}{r\partial\Theta}\right)^2 + \left(\frac{\partial\Phi}{\partial r}\right)^2. \tag{6}$$

We find the pressure exerted by the liquid on the surface of the bubble:

$$\frac{P_R}{\rho} = \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + R\frac{d^2R}{dt^2} + \left(3U\frac{dR}{dt} + R\frac{dU}{dt}\right)\frac{\cos\Theta}{2} + \frac{U^2}{8}\,(19\cos^2\Theta - 7) + \frac{P_A}{\rho} + g\,(h - A\sin\omega t - x). \tag{7}$$

The bubble is therefore acted upon by a lifting force

$$F_P = -\int_0^{\pi} 2\pi R^2 P_R \sin\Theta\cos\Theta\, d\Theta. \tag{8}$$

From Eqs. (7) and (8) we obtain

$$\frac{F_P}{\rho} = \frac{4}{3}\pi R^3 g - \frac{2}{3}\pi\frac{d(UR^3)}{dt}. \tag{9}$$

Two other forces besides F_P act on the bubble:

1) viscous drag

$$F_\mu = 6\pi_\mu RU; \tag{10}$$

2) the force binding the bubble to the capillary

$$F_\sigma = 2\pi R_0 \sigma. \tag{11}$$

The growing bubble obeys the relation

$$F_P = F_\mu + F_\sigma. \tag{12}$$

Substituting expressions (9), (10), and (11) into (12), we obtain the total differential equation

$$\frac{d^2R}{dt^2} + \frac{3}{R}\left(\frac{dR}{dt}\right)^2 + \frac{3}{R}\left(A\omega\cos\omega t + \frac{3\mu}{R\rho}\right)\frac{dR}{dt} + \frac{3R_0\sigma}{\rho R^3} + \frac{9\mu}{\rho R^2}A\omega\cos\omega t - A\omega^2\sin\omega t - 2g = 0, \tag{13}$$

which describes the growth of a spherical bubble on a vertically vibrating capillary immersed in a liquid. The solution of Eq. (13) makes it possible to determine the bubble radius R(t), the contact surface S(t) of the phases, the rate of change dS(t)/dt of that surface, and the relative velocity U of the phases at any time during growth of the bubble.

In a special case we consider the breakoff of the bubble from the capillary, at which time it ceases to grow, i.e.,

$$R = a;\quad t = T;\ \frac{dR}{dt} = 0;\quad \frac{d^2R}{dt^2} = 0.$$

Equation (13) takes the form

$$-\frac{3R_0\sigma}{\rho a^3} - \frac{9\mu}{\rho a^2}A\omega\cos\omega T + A\omega^2\sin\omega T + 2g = 0. \tag{14}$$

According to the data of [2], the breakoff of a bubble from a vibrating capillary immersed in a low-viscosity liquid takes place at the instant that the capillary moves from the uppermost point of its motion to static equilibrium with the vibration phase

$$\omega T = (4k+1)\frac{\pi}{2}\quad (k = 0,\ 1,\ 2,\ \ldots). \tag{15}$$

Substituting the breakoff condition (15) into Eq. (14), we obtain

$$a^3 = \frac{3R_0\sigma}{\rho(2g + A\omega^2)}. \tag{16}$$

If, all other conditions being equal, the capillary is at rest, we readily discern that

$$a_0^3 = \frac{3R_0\sigma}{2\rho g}. \tag{17}$$

Dividing (17) by (16), we have

$$\frac{a_0^3}{a^3} = \frac{V_0}{V} = 1 + \frac{A\omega^2}{2g}, \tag{18}$$

in agreement with the results obtained in [2] for a spherical bubble.

In addition to the three main observable regimes of bubble formation [2] (nonsteady bubble shape, fan-spraying of the gas, and steady bubble shape), there is a transition regime from a nonsteady bubble shape to fan-spraying of the gas; this regime is called bubble-train formation. In the latter (Fig. 1b) we observe the continuous formation of bubbles, each one moving in the wake of the one preceding it and separated therefrom by a film of the liquid phase. We now determine the relationship between the main parameters of the investigated system in the transition (bubble-train) regime. A spherical bubble formed in a low-viscosity liquid breaks off at the instant that the capillary is situated at the uppermost point of its path (15), so that its radius is determined by expression (16). The motion of a bubble in a low-viscosity liquid is described by the equation [3]

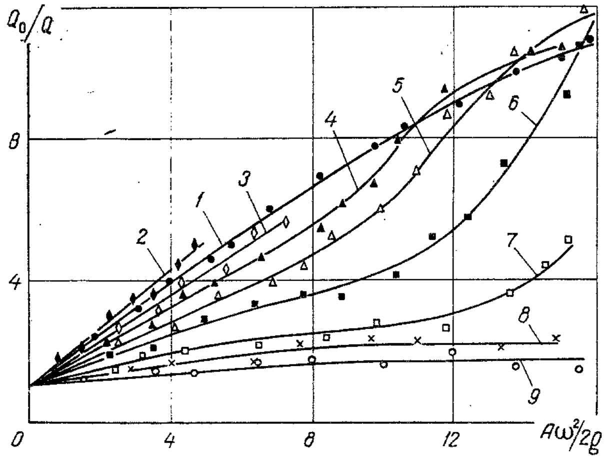

Fig. 2. Relative gas flow rate during bubble-train formation versus amplitude and frequency of vibrating capillary. 1) Theoretical relation (25); 2) experimental at 20 Hz; 3) same at 40 Hz; 4) 60; 5) 80; 6) 100; 7) 120; 8) 140; 9) 160 Hz.

$$\frac{d^2x}{dt^2} = 2g,$$

which under the initial conditions

$$\left.\frac{dx}{dt}\right|_{t=0} = 0; \quad x|_{t=0} = 0$$

has the solution

$$x = gt^2.$$

The distance from the tip of the capillary to the base of the detached bubble at time t is equal to (Fig. 1c)

$$z = x + A\sin\omega t = gt^2 + A\sin\omega t. \tag{19}$$

The condition for formation of a bubble train (Fig. 1b) is

$$z = 2a \quad \text{for} \quad t = T, \tag{20}$$

which must be satisfied together with the bubble breakoff condition (15). From (19), (15), and (20) we obtain

$$a = \frac{gT^2}{2}. \tag{21}$$

Assuming that the gas flow rate in the bubble-train regime is constant, we have

$$\frac{4}{3}\pi a^3 = QT. \tag{22}$$

Solving the system (16), (21), (22) for Q, we find

$$Q = \frac{8\pi^2 g}{9}\left[\frac{3R_0\sigma}{\rho(2g + A\omega^2)}\right]^{\frac{5}{6}}. \tag{23}$$

For $A\omega = 0$ (capillary at rest)

$$Q_0 = \frac{8\pi^2 g}{9}\left(\frac{3R_0\sigma}{2\rho g}\right)^{\frac{5}{6}}. \tag{24}$$

Dividing (24) by (23), we finally obtain

$$\frac{Q_0}{Q} = \left(1 + \frac{A\omega^2}{2g}\right)^{\frac{5}{6}}. \tag{25}$$

The latter expression shows that when a capillary having a given radius is set into vibration a train of gas bubbles begins to form at much lower gas flow rates. The solution (25) is supported by experimental data obtained on an apparatus described and illustrated in [2]. Air and distilled water were used for the gas and liquid. A comparison of the theoretical and experimental data (Fig. 2) shows that the model of a "rigid" spherical bubble can be used under the given conditions in the frequency interval from 20 to 80 Hz (curves 2-5). At other frequencies we observe qualitative agreement in the process of formation of gas bubbles on a capillary immersed in a liquid and executing vertical harmonic vibrations.

NOTATION

A, ω, amplitude and cyclic frequency of capillary vibrations; t, time; R, variable radius of growing bubble; z, coordinate of bubble center; U, velocity of bubble center; Φ, velocity potential of liquid; r, θ, spherical coordinates; P_r, pressure of liquid at point with radius-vector r; P_A, atmospheric pressure; ρ, density of liquid; q, absolute velocity of liquid; h, immersion depth of capillary at equilibrium position; x, y, coordinates of fixed reference system; F_P, lift force on bubble; F_μ, viscous drag force; F_σ, bubble—capillary binding force; μ, dynamic viscosity coefficient; σ, surface-tension coefficient; R_0, radius of vibrating capillary; g, acceleration of gravity; T, bubble-formation period; a, a_0, radii of detached bubble for vibrating and nonmoving capillaries; V, V_0, volumes of detached bubble for vibrating and nonmoving capillaries; Q, Q_0, gas flow rates through vibrating and nonmoving capillaries.

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